**Rich Mathematical Tasks: Developing Multiple   
Mathematical Competencies with Group Work**

**Objectives**

The goals of this session are to:

* Discuss the various mathematical competencies that are required to “do mathematics” successfully
* Have teachers reflect on how much opportunity they are currently providing to students to develop in each competency
* Expose teachers to mathematical tasks that foster multiple forms of mathematical growth
* Present and model student group roles that allow for teachers to implement mathematical group work effectively

**Guiding Questions**

* Is our teaching practice disproportionately valuing calculations with procedures over other mathematical competencies?
* How can we increase opportunity for students to reason, work with tables and graphs, justify methods, rephrase problems, and represent ideas/concepts/patterns?
* How can we adapt rich mathematical tasks into group work that supports and involves all group members equitably?

**At a Glance**

* Discussion of six mathematical competencies.
* Pie chart activity for teachers to reflect on their ideal balance of the six competencies and the opportunities they provide to their students in their classrooms for each.
* Group activity – mathematical task. Each teacher is assigned one group role for the task.
* Presentation and discussion of different group roles and teacher roles during group work.
* Activity – task analysis. Teachers go through four tasks and identify the mathematical competencies each task supports.
* Discussion of how teachers can adapt or design problems used in classroom instruction to develop multiple mathematical competencies.

**Materials**

Handout M3 (Working with Rich Mathematical Tasks, 11 pages)

Group roles cards, printed and cut (one set per four teachers)

Printout of yearly calendars for the next two years (one per four teachers)

**Procedure**

**Set-up:** Arrange desks or tables so that teachers can sit in groups of four. Place four PD handouts at each seating group. Have group role cards pre-cut into sets of four, kept separately with yearly calendars until “Friday the 13th” activity.

**Before We Begin: Quote “being good at mathematics” (2 minutes)** (page 1)

Facilitator welcomes teachers as they arrive

* Have teachers sit in groups of four.
* Ask teachers to read and reflect on the quote on the first page of the handout.

**Introducing the Session’s Focus: Discussion of Six Mathematical Competencies (5 minutes)** (page 2)

Facilitator goes through each of the six mathematical competencies (outlined by Jo Boaler, Professor of Mathematics Education, Stanford University) to be further explored in the session. Introduce the competencies by going through each one so that each is understood.

* *Asking questions* – This is not asking clarifying questions to a teacher, but being able to ask oneself the types of questions that are inherent to mathematical reasoning, such as “what would happen if I were to \_\_\_?” or “if I am able to figure out \_\_\_\_ [piece of the problem], would it help me find \_\_\_\_?”
* *Drawing/creating pictures and graphs* – This competency is used as a means of communicating mathematical ideas so as to make sense of the mathematics (such as when organizing information into a graph).
* *Rephrasing problems* – Being able to understand or reconceptualize a problem in a way that makes the mathematical task at hand more evident. An example is understanding that a problem asking which pizza option is a better deal is really asking which pizza provides the best cost-size ratio.
* *Justifying methods* – Being able to justify a method or approach is instrumental in mathematical reasoning and helps students transfer their learning to novel problems and situations.
* *Representing ideas/concepts/patterns* – This is being able to represent a concept, pattern, or idea in multiple ways: abstractly (such as algebraically), in words, with pictures or graphs, etc.
* *Calculating with procedures* – Fluency with skills, computations, and procedures.

**Pie Charts Activity (10 minutes)** (page 3)

Facilitator asks each teacher to reflect on his or her conception of the ideal balance of the six mathematical competencies that people who successfully use mathematics in daily life would possess and to represent that balance as a pie chart (the large circle).

* Stress that there is no wrong answer.
* Have teachers shade the pie chart as indicated by the key.

Facilitator asks teachers to reflect on the opportunity that they are currently providing to students to practice/develop in each competency, and to create a second pie chart (the small circle) depicting this information.

Facilitator introduces the session’s focus:

* There is often a disproportionate amount of focus on the “calculating with procedures” competency.
* One goal of the session is to see how rich mathematical tasks can be incorporated to help also develop other mathematical competencies in students.
* We will see how group work can be implemented in a way that supports this development.

**Friday the 13th Activity (15 minutes)** (page 4)

Facilitator informs teachers that they are to work through the Friday the 13th task from Nrich Mathematics (<http://nrich.maths.org/610>). Facilitator informs teachers that they are each going to be given a “group role” that they are to follow while working through the task.

* Pass out one group role card per teacher.
* If there is a group of three, put the Facilitator/Includer and Organizer cards together.
* If there is a group of five, have two people share the Reporter role.
* Inform teachers that only the Resource Manager can leave the table or call for help.
* Have materials (copies of calendars from future years) available, but do not pass them out. Inform groups that the Resource Manager can come seek additional materials as needed.

Give teachers a couple minutes to get started. Circulate among groups to ensure that everyone is participating.

* If a teacher is being left out or working solitarily, ask the group’s Includer to make sure he/she is including everyone in the discussion.

After five minutes, call over each group’s Resource Manager for a group huddle. Inform each Resource Manager, if they have not yet done so, to think about the first day of the month and why it would always have to be the same *day* of the week (a Sunday) on months that have a Friday the 13th, regardless of year. Send Resource Managers back to their groups.

After 15 minutes, end the activity, even if not all groups have completed the task.

**Group and Teacher Roles (7 minutes)** (pages 5-6)

Facilitator has all teachers turn to the “Group Roles” page in the handout, from Jo Boaler’s *Mathematical Mindsets: Unleashing Students' Potential Through Creative Math, Inspiring Messages and Innovative Teaching*.

Go through each group role one-by-one.

* Encourage teachers to share their experiences with each role from the activity.
* Point out that the Resource Manager role allows for the teacher to hold group huddles that do not require the entire class to stop working when the teacher needs to provide additional help.
  + This role also serves the purpose of ensuring that the group *only* calls for teacher help when everyone in the group needs help.

Have teachers turn to the “Teacher Roles” page, also from Jo Boaler’s *Mathematical Mindsets: Unleashing Students' Potential Through Creative Math, Inspiring Messages and Innovative Teaching*.

* Go through each teacher role.
  + Discuss the importance of setting up a class culture of collaboration, respect, and safety.

**Task Analysis (15 minutes)** (pages 7-10)

Have teachers independently go through the four remaining tasks (from <https://www.youcubed.org/>, <https://nrich.maths.org/>, and <https://www.scoe.org/>) and reflect on which mathematical competencies each task could develop.

* Point out that this activity is based upon how teachers visualizes *themselves* teaching the task.
* Teachers do not have to complete the task, but they may wish to start each one to get a sense of the mathematics required.

Every three minutes, have teachers go on to the next task.

After 12 minutes, point out that *having students work in groups* can increase student opportunity for certain competencies (especially ”justifying methods,” “representing ideas/concepts/patterns,” and “rephrasing problems”).

**Designing Rich Mathematical Tasks (6 minutes)** (page 11)

Facilitator goes through the “checklist” on the last page, from Jo Boaler’s *Mathematical Mindsets: Unleashing Students' Potential Through Creative Math, Inspiring Messages and Innovative Teaching*. Be clear that these are not just for designing tasks, but points to consider when incorporating existing tasks into lessons.

* “*Can you present the task before teaching the method?”*   
  Allowing students to develop their own methods often leads to deeper exploration and understanding of the math. Don’t over-support; use team huddles to provide help as needed throughout the task.
* “*Can you make it visual*?”   
  Are you allowing students to communicate their understanding visually, such as via a chart/table/ graph?
* “*Low floor and high ceiling*”   
  Is the task accessible to all learners? Are there extensions that allow the problem to be taken to a more complex level for stronger students?